Roll No. .... Total Printed Pages -12

F - 1409

C.B.S. (Fourth Semester)
EXAMINATION, May - June, 2022
MATHEMATIC STREAM
ANALYSIS-II
(M-401)

Time : Three Hours] *[Maximum Marks:40*

Note: Attempt all sections as directed.

(Section-A)

(0.5 marks each)

Choose the correct/most appropriate answer and write in your answer book:

1. Directional Derivative is used to calculate the __________ of the surface z = *f* (x,y)

   (A) Gradient

   (B) Angular velocity

   (C) Slope

   (D) None of the above

P.T.O.

2. Differentiation of implicit functions is given by __________

   (A) $0=\frac{df}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}$

   (B) $0=\frac{df}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}\frac{df}{dx}$

   (C) $0=\frac{df}{dx}=\frac{\partial f}{\partial x}.0+\frac{\partial f}{\partial y}$

   (D) $0=\frac{df}{dx}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}.0$

3. If $J=\frac{\partial(u,v)}{\partial(x,y)}$ *and* $J'=\frac{\partial(x,y)}{\partial(u,v)}$ *then* $JJ'=$ ____

   (A) 0

   (B) 1/2

   (C) 1/4

   (D) 1

4. The set R^m is called the _____ of linear map $T : R^n \to R^m$.

   (A) Codomain

   (B) domain

   (C) Range

   (D) Image

5. ___________ is a surface in the Euclidean space R^3 which is defined by a parametric equation with two parameters $r : R^2 \to R^3$.

(A) Paraboloid

(B) Parametrized surfaces

(C) Cone

(D) Sphere

6. x^2-y^2= 25 in Cylindrical Coordinates

(A) r^2= 25 cos (2θ)

(B) r^2= 25 tan (2θ)

(C) r^2= 25 cosec (2θ)

(D) r^2= 25 sec (2θ)

7. Suppose X and Y are vector spaces and $A \in L(X,Y)$. The null space of A, N (A) is the set of all $x \in X$ at which ____

(A) Ax = b (constant)

(B) Ax=0

(C) Ax = 1

(D) Ax $\neq$ 0

8. The Inverse function theorem states that a ____________ f is invertible in a neighbourhood of any point x at which the linear transformation f '(x) is invertible.

(A) Continously differential mapping

(B) Differential mapping

(C) Continous mapping

(D) Discontinous mapping

9. At a critical point, the gradient ∇f is __________

(A) One

(B) Non-zero

(C) Zero

(D) ½

10. All the critical points of f (x,y)= x sin y is ______________

(A) absolute maxima

(B) local minima

(C) saddle points

(D) local maxima

11. Among all rectangular boxes with fixed surface area of 10 square meters, there is a box of largest possible volume. Then x (length) is __________

(A) $\sqrt{2/3}$

(B) $\sqrt{1/3}$

(C) $\sqrt{5/3}$

(D) $\sqrt{7/3}$

12. A point that is either a local maximum or minimum point is called _____________

(A) global minimum

(B) global maximum

(C) a extremum

(D) a local extremum

13. The value of $\iint_R (1-x)dxdy$ where the region R is [0,1] x [0,1] is ______

(A) 1/2

(B) 1/3

(C) 1/7

(D) 1/4

P.T.O.

14. The volume of the ball $x^2 + y^2 + z^2 \leq R^2$ using spherical coordinates is _________

(A) $4\pi R^3 / 2$

(B) $4\pi R^3 / 3$

(C) $4\pi R^3 / 5$

(D) $4\pi R^3 / 7$

15. Let $f : [a,b] \to R$ be bounded with $a<b$. Say that f is Darbouxintegrable on $[a,b]$ if

(A) $L(f) < U(f)$

(B) $L(f) > U(f)$

(C) $L(f) = U(f)$

(D) $L(f) \leq U(f)$

16. If $f(x)$ is continous on $[a, \infty)$ then ________________

(A) $\int_a^{\infty} f(x)dx = \lim_{b\to\infty} \int_a^{b} f(x)dx$

(B) $\int_a^{\infty} f(x)dx = \lim_{b\to\infty} \int_a^{-b} f(x)dx$

(C) $\int_a^{\infty} f(x)dx = \lim_{b\to-\infty} \int_a^{b} f(x)dx$

(D) $\int_a^{\infty} f(x)dx = \lim_{b\to-\infty} \int_{-a}^{b} f(x)dx$

17. R=xI+yJ+zK then $\nabla.R =$ _____________

(A) 1

(B) 2

(C) 3

(D) 0

18. When the circulation of F around every closed curve in a region E vanishes, F is said to be __________________

(A) Normal in E

(B) Solenoidal in E

(C) Irrotational in E

(D) Smooth in E

19. Green's Theorem in a plane is a special case of __________

(A) Divergence Theorem

(B) Stokes Theorem

(C) Gradient Theorem

(D) Scalar Theorem

20. The ___________ to R^n at P is the set of pairs $T_pR^n = \{(p,v): v \in R^n\}$

(A) Constant vector field

(B) Vector field

(C) Tangent space

(D) Contangent space

(Section-B)

(0.75 marks each)

Answer the following very short answer type questions in 2-3 sentences each :

1. State Taylor's theorem in several variables.

2. Compute the directional derivative of function $f(x,y)=x^2+y^2-3xy^3$ at pt. $(x_0,y_0)= (1,2)$ in the direction of the unit vector parallel to the given vector $v = \left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)$

3. Obtain the equation of the tangent plane in parametrized surface.

4. Write about Spherical Coordinate System.

5. Analyze the critical point at the origin for $g(x,y) = x^2+3xy+y^2$

6. Write about Lagrange's Multiplier Method in space.

7. Express (a) the surface $xz = 1$ and (b) the surface $x^2+y^2-z^2 = 1$ in spherical coordinates.

8. For what values of p does the integral $\int_1^\infty \frac{1}{x^p}dx$ converge?

9. Write about Differential Forms of Gradient, Divergence and Curl.

10. If $A = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$; find $\nabla.A(\text{div}A)$ at the (1,-1,1)

(Section-C)

(1.25 marks each)

Note- Answer the following short answer type questions in ≤ 75 words:

1. Expand $f(x,y) = x^y$ in powers of (x-1) and (y-1) and (y-1) upto the third degree terms.

2. If $x = uv, y = \frac{u+v}{u-v}$, find $\frac{\partial(u,v)}{\partial(x,y)}$

3. Show that the graph of a real valued function $f(x,y)$ is the image of a parametrized surface.

4. Write about Spherical coordinate System.

5. Find the critical points of the functions $f(x,y) = x^2 - y^2 + xy + x - y$

6. Define Maxima and Minima for the functions of two variables with examples.

7. Evaluate $\iint_D (x^2 - y^2)dxdy$ where D is the square with vertices (0,0), (1,-1), (1,1) and (2,0) using the change of variables x=u+v, y=u-v

8. Evaluate $\iint_A xy\ dxdy$, where A is the domain bounded by x-axis, ordinate x=2a and the curve $x^2 = 4ay$.

9. Verify Green's theorem for $\int_c [(xy+y^2)dx + x^2dy]$, where C is bounded by $y=x$ and $y=x^2$.

10. If $F = grad(x^3+y^3+z^3-3xyz)$, find div F?

(Section-D)

(2 marks each)

Note- Answer the following long answer type questions using 175 words:

1. Write about Partial Derivative.

OR

Write about Total Derivative.

2. Write about Cylinders.

OR

Write about Spheres.

3. Write about Saddle Point.

OR

A rectangular box, open at the top, is to hold 256 cubic centimeters of sand. Find the dimensions for which the surface area (bottom and four sides) is minimized.

4. Write about Riemann Integral? Prove that the constant function f(x)=1 on [0,1] is Riemann Integral.

OR

Write the properites of Double Integral.

5. State and prove Stoke's Theorem.

OR

Find the integral of $F(x, y, z) = z\hat{i} - x\hat{j} - y\hat{k}$ around the triangle with vertices (0,0,0), (0,2,0) and (0,0,2) using stoke's theorem.